

Shepherd's Necklace

1. A small shepherd boy was responsible for taking to pasture each day 23 sheep. These sheep would graze freely all day, and in the evening the boy was to round them all up and drive them home. Unfortunately, the boy could not count, so he had difficulty each day deciding whether he had found all the sheep before returning home.
 - a. Can you devise a method (other than teaching the boy to count) that would make it possible for him to be sure he had them all? If so, describe it.
 - b. The sheep's owner came up with a simple scheme. It involved making, for each sheep, a string necklace that would fit around the sheep's neck. In the morning, the boy would take the necklaces and place them all around his own neck. What should he do with them in the evening?
 - c. The "necklace idea" spread throughout the region and soon all the young people tending sheep went to the meadows with string necklaces around their necks. One day, an argument arose between a shepherd and a shepherdess as to which was tending the largest number of sheep. Resolving the argument was made difficult since neither could count. Can you describe a method by which they could resolve their argument while leaving their sheep grazing undisturbed?
2. One day an argument arose between two four-year-olds as to who had the most Lego pieces. They each counted; one reported having a "million and five" while the other reported "a thousandy hundred". Needless to say, a recount revealed that neither could reliably count high enough to count the Legos belonging to either of them. How can they settle their argument without the intervention of adults?
3. Consider the following game. We deal an ordinary deck of 52 cards, and each of us gets 26 cards. We simultaneously turn up a card. If both cards are black, you get them. If both cards are red, I get them. If they are different colors, they are discarded. The process is repeated until all the cards are used. You get \$5 if you have won more cards. I get \$1 if you haven't. Is it profitable for you to play the game with me? Explain.